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Senior Thesis

Statistical and Dynamical Properties of a Vibrated Granular Polymer

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Contents

1 Introduction

Granular polymers consist of particles connected to each other flexibly by rods to form a chain. There has been interest recently in the motion of granular polymers when excited by external forces, such as from collisions with other particles in the environment. While there are many analytical models to predict these properties, there are only a few recent studies that experimentally test them on macroscopic granular polymers.

In this case, chains were driven into motion by a vibrating plate that they sat on. Steel chains were used as polymers, each consisting of flexibly linked three millimeter steel beads. With high speed camera and a particle identification program, data was collected on the two dimensional movement of these chains. Chain length and acceleration of the vibrating tray were varied, and statistics were taken on radius of gyration, end-to-end separation, the motion of the center of mass, and temperature. Results are compared to the predictions from a model of a self-avoiding random walk.

2 Self-Avoiding Random Walk Model

Polymers are theorized to behave as the self-avoiding random walk model describes. In two dimensions, this model constructs a granular polymer of length N by starting with one particle and adding to it. The second particle is ran-

domly placed a distance d away from the first. Likewise, the third particle is randomly placed a distance d away from the second particle, but it can not take up any of the space occupied by the first particle. The remaining N-3 particles are placed randomly on a circle of radius d around the previous particle, following the conditions that no beads can overlap, and the chain is not allowed to cross itself.

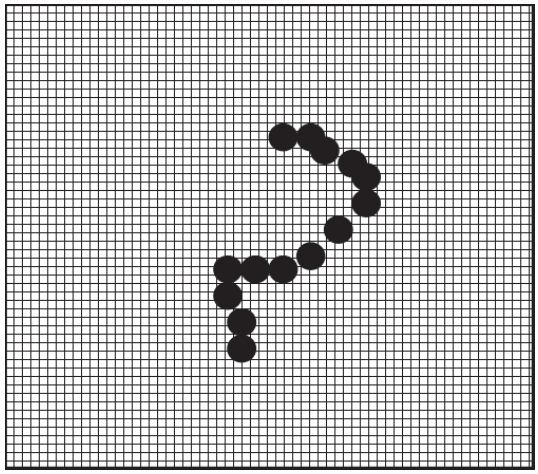

Figure 1: Self-avoiding polymer.

This study is interested in the model's predictions of the characteristic size (R), diffusion (D), and temperature (T) of self avoiding polymers. In two dimensions, the prediction is:

$$R \propto N^{\nu} \quad \text{where} \quad N >> 1$$

$$\nu = 3/4 \quad \text{(for self-avoiding chains)}$$

$$\nu = 1/2 \quad \text{(for ideal chains)} \tag{1}$$

Diffusion, or the variance in the position of the center of mass over time, will follow:

$$D \propto T/N^{\epsilon} \quad \text{where} \quad \epsilon = 1.0 \tag{2}$$

The temperature is the variance in the position of the individual particles in the polymer over time, and is predicted to be:

$$T \propto \Gamma \quad \text{and} \quad T \not\propto N \tag{3}$$

where Γ is the acceleration of the shaker. [3]

These three characteristics were measured experimentally in a slightly different situation. The model applies, for example, to a perfume polymer in air. This polymer would be exited in a continuous manner by other molecules hitting it randomly. In the case of a vibrated chain, the excitation is periodic, with the vibrating tray exciting all of the beads simultaneously at regular intervals (30Hz). The model also assumes elastic collisions, while the macroscopic model studied here is clearly excited through inelastic collisions.

3 Related Studies

There have been a few recent studies measuring the properties of granular polymers in similar situations. One study measured the properties of relatively short polymers (up to 15 beads) driven by a granular fluid made up of self-propelled motorized balls [1]. Their conclusion for the end-to-end separation of the polymers was $\nu = .75 \pm .01$. Diffusion and temperature are not discussed in this

paper. Our study differs most significantly in that the method of agitation is vibration instead of self-propelled balls and much longer chain lengths are used (up to 128 beads).

Another study used the same particle identification process on the same kind of vibrating steel beaded chains, but in the case where the ends were connected to form looped chains [2]. This study reports a bead identification accuracy rate of better than 96% due to extra identified particles from unwanted reflections off the connectors between beads, and insufficiently dim reflections off the seams of beads. Those problems were initially experienced, but overcome in our study by shining light from several sources through thin paper to illuminate the chains from a diffused light source.

4 Experimental Setup

Eight different chain lengths were used (N= 2^i where i = 1, 2, 3, ..., 7 and N= 96). Each chain was composed of 3mm hollow steel balls connected flexibly with steel connectors. The beads are shiny and reflect light well (this is important for bead identification).

A chain was placed onto a circular tray with a lip which has small glued down beads (d = 1mm) covering its surface (including lip). The entire surface of the tray including its sides are roughened in this way so that when vibrated, collisions between chains and the tray result in horizontal as well as vertical

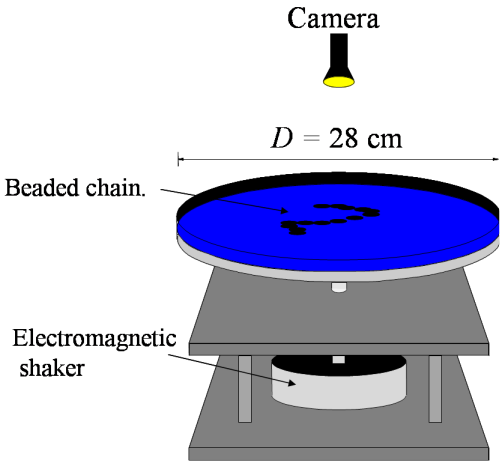

Figure 2: Experimental Setup

motion. The tray is vertically oscillated by a shaker (constructed by Daniel Blair [4]) which receives signal and power from a function generator and power supply. An oscilloscope is also hooked up to the shaker so that oscillations can be directly monitored. The acceleration of the shaker was adjusted until it was as high as possible without actually causing the chains to move, or lose contact with the tray (which could be determined by looking for spikes on the oscilloscope). Then this acceleration was doubled ($\Gamma = 2$gs) and tripled ($\Gamma = 3$gs). The frequency was set to 30Hz.

Lighting was provided by four lights, positioned evenly around the tray and approximately 30 degrees above surface of the tray. The lights shined through thin paper, so as to light the chains from as many angles as possible and provide roughly circular reflections off the top of each bead. A camera located approxi-

mately a meter directly above the center of the tray was used to record images of these bright circular reflections which contrasted well with the dark beads covering the surface of the tray. By adjusting the focus of the camera so that images were slightly blurry, the reflections appeared more solid and circular. Images were recorded every 50ms for 150s; four sets of images were taken for the shorter chain lengths, eight for the larger (N$>$ 32). The eighth videos for the largest chain lengths (N$=64, 96, 128$) were taken at a 2 frames per second so that videos would be $\sim$25 minutes long (longer chains move about much more slowly).

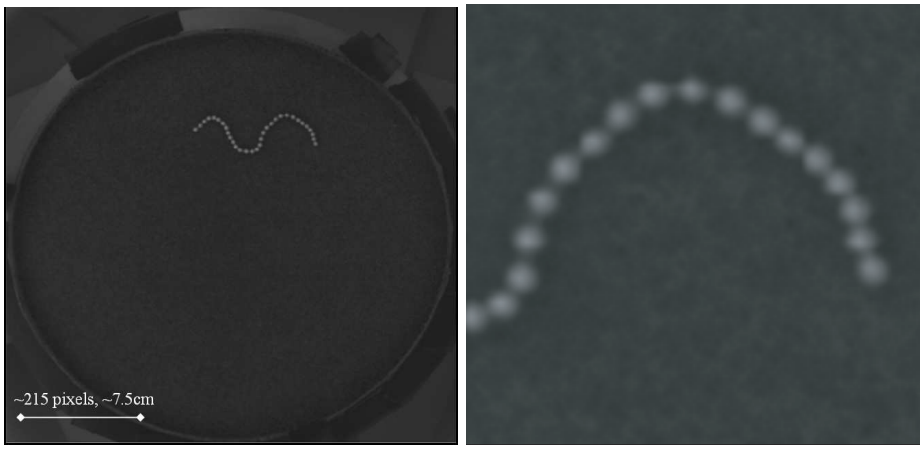

Figure 3: Image and blowup of 32 bead long chain. One bead diameter is approximately 6-8 pixels.

Particle Identification

Images were transfered to a computer and scanned to find clusters of light pixels. The clusters were identified as beads if they were roughly circular, within

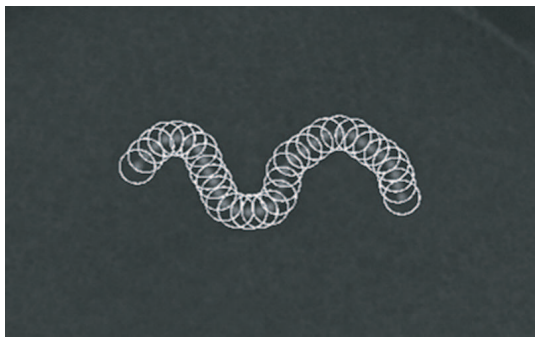

Figure 4: Identified beads are circled.

a specified diameter range, and within a specified brightness range. The center of each identified bead was specified to sub-pixel accuracy (for example, a bead's position might be identified as between two pixels). This process worked extremely well, with more than 99.9% of the beads correctly identified, and no false identifications. Occasionally when a bead was shadowed from one of the lights by the lip of the tray its reflection would not be bright enough to be identified.

5 Static Properties

Characteristic polymer size is a static property that was measured experimentally. It was measured both by calculating the radius of gyration and the average end-to-end displacement. The prediction of the self-avoiding random walk model (from equation 1) are $R \propto N^{\nu}$ where $\nu = 3/4$.

5.1 Radius of Gyration

The radius of gyration is calculated using the following formula:

$$R_g^2 = 1/N \sum[(R_n - R_{cm})^2] \quad \text{where} \quad R_{cm} = 1/N \sum_{n=1}^{N} R_n \tag{4}$$

The source code used to calculate the radius of gyration is available in Appendix A.

In words, the radius of gyration measures how far apart the individual particles of a granular polymer are from its center of mass. The calculation of the average radius of gyration was straightforward given the identified bead positions from each image. By plotting a best fit line an experimentally determined exponent that describes the relationship between polymer length and radius of gyration can be determined. Ignoring $N = 2$ and $N = 4$, (because the model assumes $N >> 1$) this exponent is found to be $\nu = 0.729$.

Looking at the figure 5.1, it appears as though the results from the longest chains are not as close to the best fit line as they should be. This could be a result of the boundary conditions due to the sides of the tray. If the data from these last two chain lengths are ignored (in addition to ignoring the first two chain lengths), then the exponent becomes $\nu = 0.808$, which is now higher than predicted.

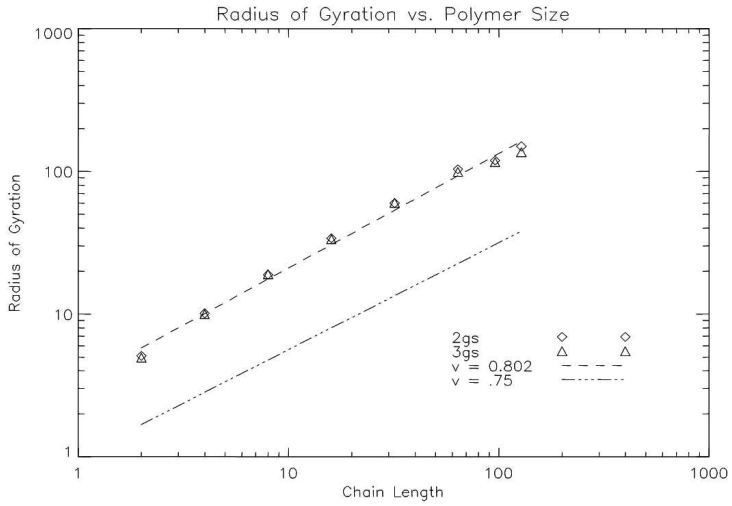

Figure 5: Radius of gyration results indicate ν is larger than expected.

5.2 End-to-End Separation

In order to measure the average end-to-end separation, the chain ends had to be identified in as many frames as possible. This was accomplished by counting how many neighbors were located within a radius of each bead. If a bead only had one neighbor, it was identified as an end, and if there were exactly two identified ends in an image, the end-to-end distance was recorded. The source code used to do this is available in Appendix B. This measurement was not possible in 8.89% of the images. Most of the time when this method failed it was due to the case when an end of the chain ran into one of the beads in the center of the chain (as illustrated).

The experimental results gave an exponent of $\nu = 0.625$. This is significantly

smaller than the predicted slope of $\nu = .75$. Again, $N = 2$ and $N = 4$ were ignored. By ignoring the largest chain lengths for the same reasons as in the radius of gyration, the exponent became $\nu = 0.749$, almost exactly as predicted.

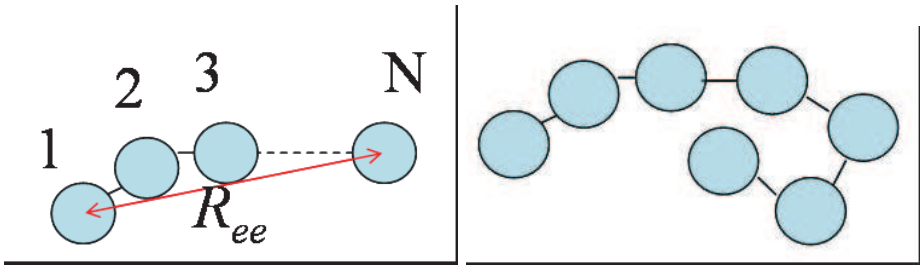

Figure 6: End-to-end separation. Right image displays case where only one end can be detected.

6 Dynamic Properties

Dynamic properties measure changes in position of the polymer over time. The diffusion of the centers of mass (COM) were determined, as well as the temperature of the individual beads in the polymers.

6.1 Center of Mass Statistics:

The center of mass (COM) can be calculated from equation 4. Once the COM was determined for every image, several useful graphs were made to describe its motion. First, to get a sense of the motion of the chain over the course of

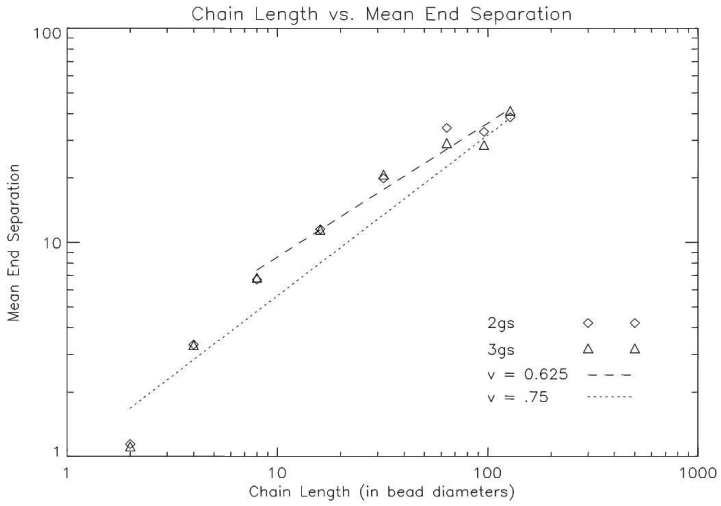

Figure 7: End-to-end separation results indicate that ν is almost exactly as predicted by the model.

150s, the x and y-coordinates of the COM were simply plotted against each other. This also served to confirm that there were no problems with the particle identification which would lead to unexpected jumps in the position of the polymer.

By plotting the time between images vs. the standard deviation of the COM's motion, two trends became apparent. As intuition suggests, diffusion is inversely proportional to polymer length, and proportional to the time between frames (t). The slopes of these lines were then used to calculate a diffusion constant (D) according to:

$$< (R_{cm}(t) - R_{cm}(0))^2 >= 2Dt \tag{5}$$

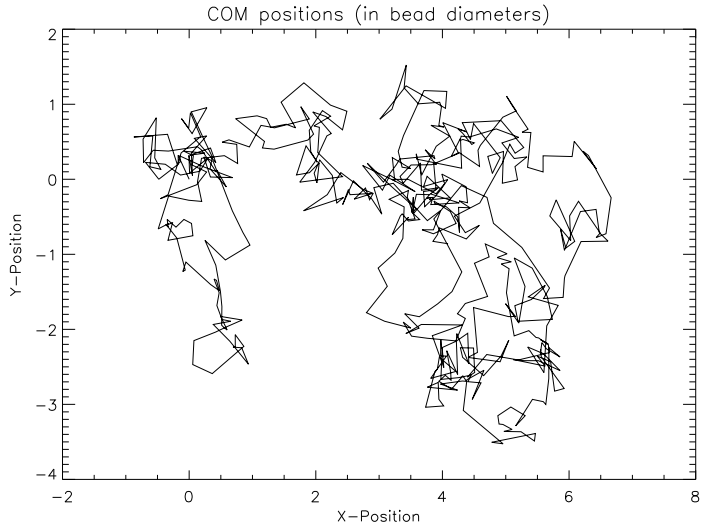

Figure 8: Positions of the center of mass of a 64 bead polymer over 150 seconds (in bead diameters).

The diffusion constant ranks how much a chain length diffuses.

By now plotting the chain length against these diffusion constants, a comparison can be made to the results predicted by the self-avoiding granular polymer model. The model predicts in equation 2 that diffusion will be $D \propto T/N^{\epsilon}$ where $\epsilon = 1.0$. According to the experimental results, $\epsilon = .964$, which clearly is very close to its expected value. It's important to notice that this assumes that temperature is constant.

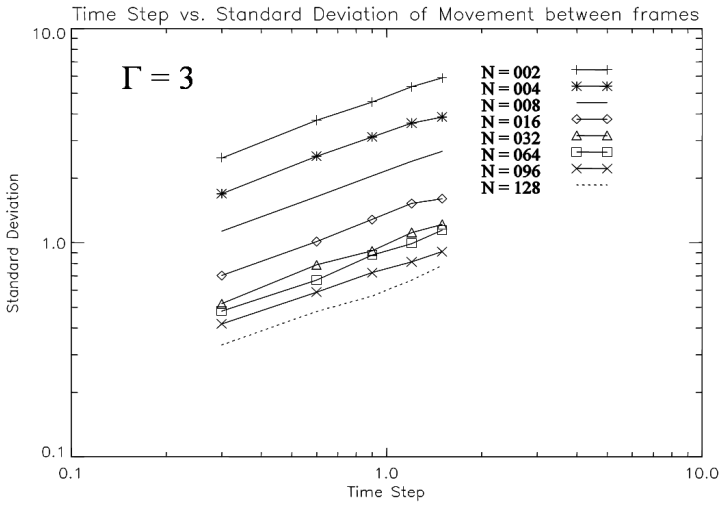

Figure 9: A diffusion constant can be determined from the slopes of each of these lines.

6.2 Temperature:

In order to calculate the temperature, the individual beads had to be tracked from one frame to the next. To accomplish this, the frame rate was increased to 400fps (one image every 2.5ms), which shortened the length of the videos to about eight seconds. Only one set of images was used for each chain length at each acceleration. A program (code available in Appendix C) was run that compared the positions of the identified particles in one frame to their positions in the next. A particle that was within a radius (smaller than a bead diameter) of the same position in the next frame was considered to be the same particle in the consecutive frames. This kind of identification over time was not always possible because of beads that were not identified, and also because even at

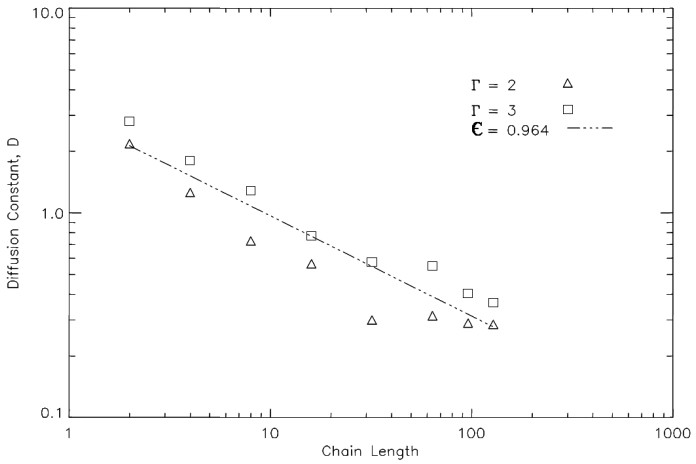

Figure 10: The slope of this line fits the prediction of the model very well.

400fps there was sometimes more than one particle in the radius of a particle in the previous frame. Only for about 5 of the frames (out of 3200 frames) in a video was it impossible to track from one frame to the next.

A plot of chain length against temperature indicates that except for the shortest chain lengths, the variance of the individual beads is independent of polymer length. The temperature does, however, depend on the acceleration. This is what the model predicted in equation 3, and this also confirms that the assumption of constant temperature when calculating the diffusion exponent was fair in the limit of $N >> 1$.

The shortest chain lengths may not fit the model because the ends of the chains have more freedom than the inner beads. In the shortest chains, the

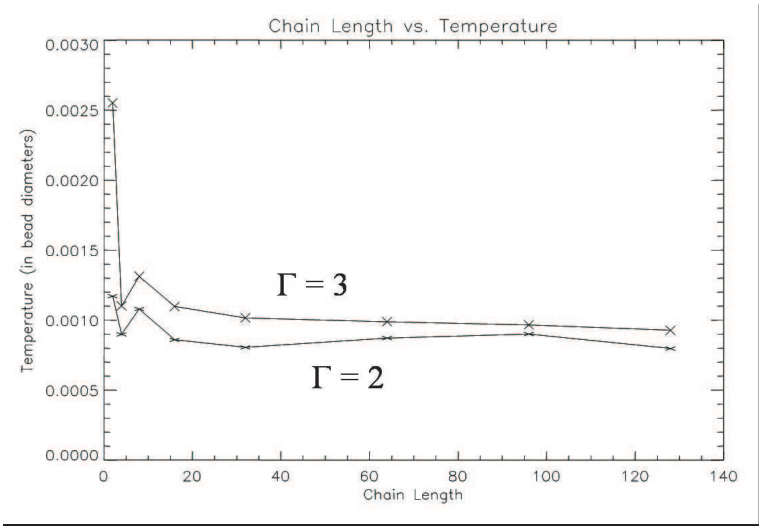

Figure 11: Temperature is independent of chain length and dependent on acceleration in the limit of $N >> 1$.

two ends are a significant portion of the beads in the chain. They are only connected on one end, and therefore are free to move further between frames. Before accepting this argument, the variance of the end beads would need to be measured and compared to the variance of the inner beads. Also, because the graph is not very clean at the shortest chain lengths, more data could help to clarify how the temperature strays from its independence from chain length.

7 Conclusion

The most important conclusion is that the properties of these macroscopic chains are very comparable to the predicted behavior in the model, despite the dif-

ferent conditions (periodic vs. continuous excitation and inelastic vs. elastic collisions). The characteristic size was shown to fit the model poorly, but if boundary conditions are assumed to play a role, this fit becomes better. Diffusion and temperature (in the limit of $N >> 1$) were shown to fit the model quite well.

A Radius of Gyration IDL code

```
for i=0, num_files-1, frame_skpd do begin
;cycles through files of current video
  fn = filenames(i)
  a = read_gdf(fn)
  num_part = size(a, /n_elements)/5
  xcom = 0
  ycom = 0
  rad = 0

  for j=0, num_part-1 do begin
  ;cycles through identified particles from current file
  ;subtract one from num_part so j starts from 0, not 1
    xcom = xcom + a(0,j);/part_diam
    ycom = ycom + a(1,j);/part_diam
    if i eq 0 then begin
      ini_xcom = xcom;/num_part
      ;stores the initial x and y COM values
      ini_ycom = ycom;/num_part
    endif
    if j eq num_part-1 then com = [i/frames_skpd, xcom/part_diam - $
          ini_xcom/part_diam, ycom/part_diam - ini_ycom/part_diam, 0]
```

```
  endfor

  for k=0, num_part-1 do begin
  ;cycles through identified particles from current file again
    rad_x = a(0,k) - xcom/num_part
    rad_y = a(1,k) - ycom/num_part
    rad = rad + sqrt(rad_x^2 + rad_y^2)
    if k eq num_part-1 then com[3,0] = rad/num_part
  endfor

;   vid_ave_rad = vid_ave_rad + rad/num_part
  if i eq 0 then com_all_vid = com else com_all_vid = [[com_all_vid], [com]]

endfor    ;num_files

vid_ave_rad = total(com_all_vid[3,*])/(n_elements(com_all_vid)/4)
if n eq first_vid then com_all_chn = com_all_vid else com_all_chn = $
                                     [[com_all_chn], [com_all_vid]]
tot_ave_rad = total(com_all_chn[3,*])/(n_elements(com_all_chn)/4)

if n eq num_vids and m eq smllst_chn then r_of_g = [m, tot_ave_rad] $
else if n eq num_vids then r_of_g = [[r_of_g], [m, tot_ave_rad]]
;r_of_g stores [chain length, radius of gyration]
```

B End-to-End Separation IDL code

```
for i=0, num_files-1 do begin

  fn = filenames(i)

  a = read_gdf(fn)

  num_ends = 0

  for j=0, size(a, /n_elements)/5 -1 do begin

    neighbors = 0

    for k=0, size(a, /n_elements)/5 -1 do begin

      dist = sqrt((a(0,j) - a(0,k))^2 + (a(1,j) - a(1,k))^2)

      if dist lt 1.82*part_diam and dist ge 1 then begin

        neighbors = neighbors +1

        ends = [a(0,j), a(1,j)]

      endif

    endfor     ;k

    if neighbors eq 1 then num_ends = num_ends +1

    if neighbors eq 1 and num_ends eq 1 then both_ends $

    = [i, ends(0,0), ends(1,0)]

    ;****creates end array containing chain end locations and

    ;separation distance for each frame in video***

    if neighbors eq 1 and num_ends eq 2 then begin

      end_disp = sqrt((both_ends(1,0) - ends(0,0))^2 + $

                 (both_ends(2,0) - ends(1,0))^2)
```

```
      both_ends = transpose(both_ends)
      ends = transpose(ends)
      both_ends = [[both_ends], [ends], [end_disp]]
      both_ends = transpose(both_ends)
      both_ends(0,0) = both_ends(0,0)*part_diam
      ;because it is about to be divided by part_diam
      if vid_ends(0,0) eq 100 then vid_ends = both_ends/part_diam $
      else vid_ends = [[vid_ends], [both_ends/part_diam]]
    endif
    ;[file, end1 x_pos, end1 y_pos, end2 x_pos, end2 y_pos, end displacement]
    ;units of bead diameters
    ;[0   , 1         , 2         , 3         , 4         , 5               ]
    ;******
  endfor       ;j
endfor         ;i, files
```

C Temperature IDL code

```
  for i=float(0), length-2 do begin
  if file(3,i) eq file(3,i+1) then begin
    line = [file(0,i)/bead_diam, file(1,i)/bead_diam, $
    (file(0,i) - file(0,i+1))/bead_diam, (file(1,i) - file(1,i+1))/bead_diam]
      if i eq 0 then array = line else array = [[array], [line]]
```

```
     endif
  endfor   ;i


;print, max(array[2,*]), min(array[2,*])
max = max(array[2,*])


;print, file[3,length-1]


bin_array = make_array(2,200, /float, value=0)
for i=0, 199 do bin_array[0,i] = i-100
for j=float(0), length-file[3,length-1]-2 do begin
  bin_array[1, ceil(array[2,j] * 100) + 100] = $
    bin_array[1, abs(ceil(array[2,j] * 100)) + 100] + 1
endfor


;print, bin_array
plot, bin_array[0,*]/100, bin_array[1,*]/100


temp = variance(array[2,*])
print, temp
```

References

[1] Prentis, Jeffrey and Sisan, Daniel, *Granular Polymer Solution,* University of Michigan at Dearborn, MI, 2002

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[4] Blair, Daniel L., *Statistical Analysis of Granular Gases, Pattern Formation, and Crumpling through Real Space Imaging,* Clark University, MA, 2003